Technical Comments

Remarks on the Vortex Model of Wing-Body Interference

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WITHIN the framework of slender body theory, wing-body c mbinations of complicated configurations may be analyzed approximately by the vortex method.[1,2] In such an approach to the interference problem, the body is assumed to be a circular cylinder represented by a doublet, and the wing panels are represented by a finite number of vortices. The doublet and the vortices satisfy the linear Laplace equation in the cross-flow plane. It remains to satisfy the boundary conditions. The condition of a vanishing normal velocity component on the body is satisfied by the image vortices in the circle. For a given number N of vortices for each wing panel, we have at our disposal the strength and the spacing of each vortex, which are to be determined by further restraining conditions. The purpose of the present note is to point out the correct image system and to recommend the appropriate conditions for the vortex model.

For a vortex of strength γ at the point $z = x + iy$ outside the circular cylinder $|z| = a$, the boundary condition is satisfied by the method of images, which consists of a vortex of strength $-\gamma$ at the inverse point $a^2\bar{z}$ and a vortex of strength γ at the center. This statement can be rigorously proved by the circle theorem of Milne-Thomson.[3] The image system most commonly used[1,2] consists of only the vortex at the inverse point, but not the vortex at the center. Although both systems satisfy the boundary condition of vanishing normal velocity on the circle, the latter system, unfortunately, does not obey the Kelvin theorem on the constancy of circulation. If the original vortices outside the circle add up to zero in strength, the vortices at the center cancel out. Thus, neglect of the center vortex is applicable only to symmetric configuration for which the sum of external vortices is zero. In general, it is important to include the image vortex system at the center. For example, in a recent work of Barnes,[4] the two trailing vortices outside the cylinder do not cancel each other; the correct image system should include a vortex of strength equal to the sum of the two external vortices at the center as well as two vortices at the inverse points. However, Ref. 4 did not have the vortex system at the center, the effect of which would be of interest.

As mentioned previously, the strengths and the locations of wing vortices are at our disposal. Consequently, there is certain arbitrariness in the vortex method. To determine these quantities, Nielsen suggested the following conditions: 1) the sum of the strengths of the wing vortices equals the circulation at the wing-body juncture; 2) the centroid of the vortices remains at rest. For N discrete vortices, there are $2N$ unknowns; these two conditions are certainly not sufficient, unless $N = 1$. Besides, the circulation at the wing-body juncture and the centroid of the vortices are usually not known in advance.

The number of unknowns is halved if one arbitrarily specifies the strength or location of each of the N vortices. The remaining N unknowns are determined from no-flow boundary conditions across the wing panel at N control points. For a given geometry, it is convenient to specify the vortex locations as being evenly distributed over the exposed wing span. The control points may be located midway between two neighboring vortices. This method is suggested by Campbell[1] and has been found convenient for machine computation when N is large.

Once the strengths of vortices at the specified locations are solved, the aerodynamic loading, normal force, side force, and rolling moment may be readily evaluated.

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[3] Milne-Thomson, L. M., *Theoretical Hydrodynamics* (The Macmillan Co., New York, 1960) 4th ed., Chap. XIII, pp. 350 and 362.

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Addendum: Real Solutions of Sets of Nonlinear Equations

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IT has just come to the attention of the author that the method described in the aforementioned Note is discussed by M. N. Yakolev in a 1964 paper entitled "Solutions of systems of nonlinear equations by a method of differentiation with respect to a Parameter," a translation of which appears in USSR Computational Math., **4**, no. 1, 198–203 (1964). In this paper, two references to earlier work on this subject are cited: Kiriya, V. S., "Motions of bodies in a resisting medium," Tr. Tbilissk. Gos. Univ. **44**, 1–20 (1951) and Davidenko, D. F., "An approximate solution of systems of nonlinear equations," Ukr. Mat. Sh. **5**, 196–206 (1953).

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